Example 5: A rectangular storage container with an open top has a volume of $20m^3$. The length of its base is twice its width. Material for the base costs Rs 10 per square meters, material for the sides costs Rs 4 per square meters. Express the cost of materials as a function of the width of the base.

Solution:

Let ℓ, b, h are length, width and height of the rectangular box. Then the volume of the box is,

$$V = \ell bh$$

Given that the volume of the box is $20m^3$. So,

$$\ell bh = 20 \qquad \ldots (i)$$

Since the box is with an open top. Then the surface area of the box is,

$$A = 2(\ell h + bh) + \ell b \qquad \ldots (ii)$$

Given that the base is Rs. 10 per square meter and of the sides is Rs. 4 per square meter. So, total cost is,

$$T = 8(\ell h + bh) + 10\ell b \qquad \ldots (iii)$$

Given that the length is twice of the width. So,

$$\ell = 2b \qquad \ldots (iv)$$

Now, (iii) becomes,

$$T = 10(2b)\, b + 8(\ell + b)h \qquad [\because \text{using iii}]$$

$$= 20b^2 + 8(2b + b)\left(\frac{20}{2b^2}\right) \qquad [\because \text{using (i) and (iv)}]$$

$$= 20b^2 + 80\left(\frac{3b}{b^2}\right)$$

$$= 20b^2 + \frac{240}{b}$$

which is the total cost.